Intuitions for Multiplication in Amazonian Adults and in U.S. Adults and Children

Véronique Izard[1], Pierre Pica[2], Stanislas Dehaene[3,4,5,6], Elizabeth Spelke[1]

[1] Psychology Department, Harvard University, Cambridge, MA 02138, USA; [2] Unité Mixte de Recherche 7023 « Formal Structure of Language », CNRS and Paris VIII University, Paris, France; [3] INSERM, Cognitive Neuro-imaging Unit, IFR 49, Gif sur Yvette, France; [4] CEA, NeuroSpin center, IFR 49, Gif sur Yvette, France; [5] Univ Paris-Sud, IFR49, F-91191 Gif/Yvette, France; [6] Collège de France, Paris, France.

INTRODUCTION

Success at non-symbolic addition and subtraction has been documented with diverse populations: preschool children, occidental adults (in conditions preventing counting), and also an indigene group from the Amazon, the Mundurucu, who speak a language with a restricted number lexicon (Barth et al., 2005; Pica et al., 2004). In all these populations, performance shows a characteristic ratio effect: the sums and differences that participants estimate are approximate, and the range of uncertainty increases for larger numbers.

Studies of brain-damaged patients (Lemer et al, 2003), brain imaging (Dehaene et al., 2003), or task interference (Lee & Kang, 2002) suggest that **multiplication and division, on the contrary, may be mediated by language and rote-learned tables**. Nevertheless, the existence of a universal capacity for nonsymbolic multiplication has not been investigated directly.

Here, we investigated whether multiplication is part of humans' universal knowledge about numbers. We presented computer animations illustrating non-symbolic multiplication problems to three different populations: US adults, Amazonian Adults, and US children at two ages, before and during the first years of learning multiplication (respectively Preschool-1st grade, and 3rd-5th grade).

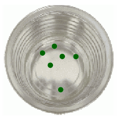
DESIGN, BASIC RESULTS

	age	Number of trials	Product sizes (n1*n2)	Comparison ratios (n1*n2/n3)	Comparison to Chance (T)	Effect of comparison ratio (ANOVA)	No effect of product size (ANOVA)
US adults (N=16, 9 females)	34.3y (19-58)	96	**36**: 4*9, 6*6, 9*4 **49**: 5*10, 7*7, 10*5 **64**: 6*11, 8*8, 11*6 **81**: 7*12, 9*9, 12*7	1.2, 1.5, 2.0	82.3%, P<0.0001	P<0.0001	P=0.081 (Fluctuations; no systematic dependency)
Mundurucu adults (N=12, 10 females)	40.8y (17-75)	48	**36**, **49** (cf above)	1.2, 1.5, 2.0	70.8%, P=0.00028	P=0.029	F<1
US Preschoolers, Kindergardeners and 1st graders (N=18, 11 females)	6.7y (5.0-7.6)	12	**36**, **49** (cf above)	1.2, 1.5, 2.0	72.7%, P=0.00014	F<1	F<1
US 3-5th graders (N=16, 11 females)	9.8y (8.1-12.2)	12	**36**, **49** (cf above)	1.2, 1.5, 2.0	81.3%, p<0.0001	P=0.071	F<1

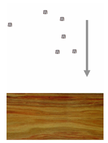
All groups performed well above chance. Except for the younger US children, performance followed the same pattern as in the addition/subtraction tasks: effect of comparison ratio, no effect of product size.

This first analysis suggests that intuitions for multiplication are universal (in the sense that they emerge spontaneously from a certain age), as is the case for addition/subtraction. As a second step, in the following analyses, we investigated whether all groups were really performing the task in the same way.

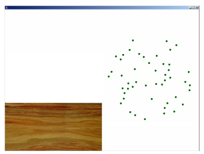
DISPLAYS



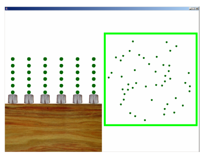
1. *Look, in this bucket, I have some seeds* [**n1**].



2. *In fact, I have many buckets* [**n2**], *and they all have the same quantity of seeds.*
I hide them all here in a big wooden box.



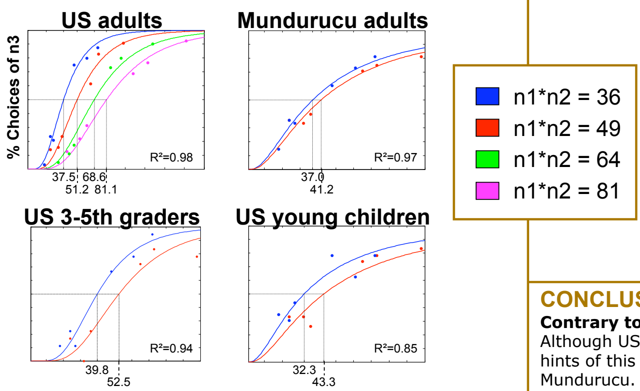
3. *Here, there are some more seeds* [**n3**].
Where are there more seeds: inside the box [n1*n2], *or outside the box* [n3]*?*



Feedback. *Good job!*

Could participants have responded based on the range of the probe (n3) alone, instead of computing the multiplication?

If participants are truly multiplying, then their choices of n3 will be modulated by the size of n1*n2.



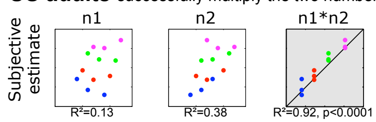
Likelihood analyses[1] indicate that the responses are better modeled by several responses curves vs. only one in all US groups : **responses were modulated by the size of the multiplication problem presented.**
The subjective equivalence point (50% of choices on either sides) read from the fits correspond the subjective estimate of the number of seeds in the box.

1. (Aikike's Information Criterion, corrected for the number of parameters in the model and the sample size – Glover & Dixon, 2004)

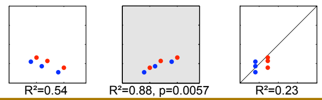
How did participants use the numeric information about n1, n2? Were they really multiplying?

Extending the previous analysis, we computed the subjective estimates of the operation separately for the different decompositions of the products (eg 4*9 vs. 6*6 vs. 9*4). The following graphs depict these estimates in function of n1 alone, n2 alone, or n1*n2.

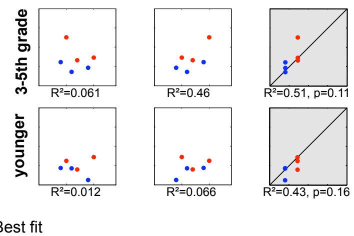
US adults successfully multiply the two numbers.



Mundurucu adults used the number of boxes, and ignored the number of seeds per box.



US children showed a weak competence for non-symbolic multiplication?



CONCLUSION

Contrary to addition, intuitions for multiplication are not universal in humans. Although US Adults can multiply sets of numbers quite accurately, and US children show hints of this ability, this competence is not fully present in another culture, the Mundurucu. Instead of performing a mental multiplication on two numbers, Mundurucu participants approximated the multiplication based on only one of the two operands given, which they scaled by a constant factor. These results highlight both the limits and the possibilities of intuitive arithmetic: **Multiplication of two numbers is not universally present in humans, but scaling by a constant factor seems to be** (Barth, 2008).

In occidental populations, intuitions about multiplication may start developing even prior to formal instruction with multiplication. Therefore, the Mundurucus' limitation with multiplication may not be related to a lack of instruction with multiplication per se, but to more basic differences pertaining to the representation of sets.

Acknowledgements. This work is part of a larger project on the nature of quantification. It is based on psychological experiments and linguistics studies in the Mundurucu territory (Para) under the supervision of PP, in accordance with Consehlo de Desenvolvimento Cientifico et Tecnologicico and the Fundacão do Indio (Funaï; Processo 2857/04). We thank C Tawé, A Ramos and C Romeiro for useful discussion and advice, and Danielle Hinchey and Amy Heberle for help with data collection in Boston. Supported by INSERM, the SHS department of CNRS (P.P.), NSF (E.S.), a McDonnell Foundation centennial fellowship (S.D.).and a travel grant from the French Ministery of Research (V.I.).

References.

Barth et al, *PNAS*, 2005.
Barth, *Cognition*, 2008
Glover & Dixon, *Psychonomic Bulletin & Review*, 2004
Lee & Kang, *Cognition*, 2002
Lemer et al., *Neuropsychologia*, 2003
Pica et al., *Science*, 2004